

LINEARIZED FLIGHT DYNAMICS OF VERTICAL LIFT-OFF HIGH SPEED VEHICLES

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Abstract

This paper discusses the linearized flight dynamics of vertical lift-off high-speed aerospace vehicles, such as missiles, sounding rockets, and satellite launch vehicles. The equations presented are particularly useful for control law design and its validation by testing through simulation. With regards to vertical lift-off high-speed vehicles the limitations of the conventional aircraft dynamics approach are discussed. One such limitation is the singularity in computing Euler angles for vertical lift-off ($\Theta_0 = 90^\circ$). It is observed that a different Euler angle sequence for attitude tracking can shift the vertical lift-off (pitch) singularity to yaw-angle, which is not a cause for concern since such vehicles are not expected to execute large lateral maneuvers. Also the proposed Euler angle sequence simplifies the perturbation relations between body-axes angular rates and Euler angle rates. This means that for small perturbations, the Euler angle rates can directly be equated to the body-axes sensed rates. Further, the linear model developed in the body axes, can accurately be used to describe the dynamics of the vehicle flying near orbital speeds over a non-rotating and spherical earth. Expressions for stability derivatives are derived. The linear time invariant system of equations can be extended to incorporate time-varying data when implemented on a computer to give a linear time-varying simulation. Comparisons with non-linear 6DOF simulation are presented.

Nomenclature

C_x	$= X_a / \hat{q}A$, axial force coefficient
$C_{y(z)}$	$= Y_a(Z_a) / \hat{q}A$, side(normal)force coefficient,
C_l	$= L_a / \hat{q}AD_{ref}$, rolling-moment coefficient
$C_{m(n)}$	$= M_a(N_a) / \hat{q}AL_{ref}$, pitching(yawing)- moment coefficient
C_{l_p}	$= \partial C_l / \partial (pD_{ref} / 2V_T)$, roll-moment damping coefficient
$C_{m_q(n_r)}$	$= \partial C_{m(n)} / \partial (qL_{ref} / 2V_T)$, pitch(yaw)- moment damping coefficient
X	= force along x -axis
$Y(Z)$	= force along $y(z)$ -axis
L	= moment along x -axis
$M(N)$	= aerodynamic moment along $y(z)$ -axis
I_{xx}	= axial moment of inertia about x -axis
$I_{yy(zz)}$	=transverse moment of inertia about $y(z)$ - axis
I_{ij}	= product of inertias for $j \neq i$
η	$= 1 - I_{xx} / I_{zz}$
U, V, W	=components along the body-axes of linear velocity of body w.r.t. inertial space.
P, Q, R	=components along the body-axes of angular velocity of body w.r.t. inertial space
D_{ref}	=reference diameter
L_{ref}	=reference length
A	=reference area
\hat{q}	$= 0.5\rho V_T^2$, the dynamic pressure

ρ	=air density
V_T	$= \sqrt{U^2 + V^2 + W^2}$
α	$= \tan^{-1}(W/U)$
β	$= \sin^{-1}(V/V_T)$
M	$= V_T / a$ (a is the speed of sound)
$\xi(\zeta_{ii})$	$= \rho A / m(\rho A / I_{ii})$
c_{\angle}, s_{\angle}	$= \cos(\angle), \sin(\angle)$
\hat{U}	=equilibrium flight velocity in the plane containing the flight velocity and gravity vectors
\Re	=radial distance of vehicle from geo- centre
$\dot{\chi}$	$= \dot{\Theta} - \frac{\dot{\hat{U}}}{\Re}$
6-DOF	=six degree of freedom

Introduction

The subject of flight dynamics is mature and well understood.^{1,2,3,4} However, most of the work discusses the flight dynamics of airplanes or rotorcraft, and problems related to vertical lift-off rocket vehicles are often not explored. In addition, the focus is on flight at relatively lower speeds, and dynamics of vehicles at near orbital speeds are not as thoroughly treated. This paper studies the linearized flight dynamics of vertical lift-off high-speed aerospace vehicles such as missiles, sounding rockets, and satellite launch vehicles. The linearized model presented here can be used not only for performance analysis but also for design of control laws and closed-loop system validation

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through simulation. The simulation of the linearized dynamics can be constructed to take into account the time-varying properties of the vehicle, and such a linear time variant (LTV) simulation can be run much faster than a detailed 6-DOF nonlinear (NL) simulation. The earth is assumed to be spherical and non-rotating.

In conventional aircraft dynamics approach, the linearized longitudinal model contains a singularity at $\Theta_0 = 90^\circ$ (vertical lift-off), because of the chosen Euler angle sequence.^{1,5} This paper suggests a different Euler angle sequence for tracking the attitude of the vehicle with respect to an earth fixed reference frame. This new Euler angle sequence not only shifts the singularity in pitch to yaw, but also leads to a simplified relation between the Euler angle rates and the sensed body-axes angular rates.

Furthermore, to accurately capture the dynamics of vehicles flying near orbital speeds, the angular velocity of the vertical has been taken into account, thereby making the linearized model workable for high speed flight over a non-rotating and spherical earth. The treatment follows closely that given in Ref. [2]. Stability derivatives are also provided in body axes and simplifications for certain cases are discussed.

Finally, as an example, flight of a sounding rocket is considered. Stability derivatives and vehicle properties are computed for the chosen flight conditions, and LTV simulations are performed. Results are compared with a NL 6-DOF simulation to show the authenticity of the LTV model.

Non-linear System of Equations

Coordinate Systems

Two orthogonal coordinate systems are used to define the vehicle's position and angular orientation in space. One is the inertial coordinate system x_E, y_E, z_E , which is fixed to the earth (this can be termed inertial since we have assumed that the earth is non-rotating) in such a way that the z_E -axis is vertically downward in the direction of gravity and x_E and y_E axes lie in the horizontal plane. The second is the body fixed coordinate system x, y, z which is fixed with respect to the vehicle's c.g., in such a way that the x -axis is coincident with the vehicle centerline and is positive in the forward direction. The y -axis is along the lateral direction and z -axis is downward normal to the xy -plane.

Euler Angle Transformation

The vehicle's attitude with respect to the earth can be defined by three Euler angles Θ (pitch angle), Ψ (heading angle or yaw angle) and Φ (roll

angle). These angles can be used to determine a transformation between inertial- and body-axes. In this paper the following Euler angle sequence has been adopted: 1) Pitch (Θ), 2) Yaw (Ψ), and 3) Roll (Φ). Thus, the transformation matrix between inertial- and body-axes becomes

$$B = [\Phi][\Psi][\Theta]$$

$$= \begin{pmatrix} c\psi c\theta & s\psi & -c\psi s\theta \\ -c\phi s\psi c\theta + s\phi s\theta & c\phi c\psi & c\phi s\psi s\theta + s\phi c\theta \\ s\phi s\psi c\theta + c\phi s\theta & -s\phi c\psi & -s\phi s\psi s\theta + c\phi c\theta \end{pmatrix}.$$

As we shall see, this particular choice of Euler angle sequence avoids the pitch angle singularity in the computation for the vertical lift-off, and shifts it to yaw angle. Defining directions along body and inertial-axes as i, j, k and i_E, j_E, k_E , respectively, we can write

$$\begin{pmatrix} i \\ j \\ k \end{pmatrix} = B \begin{pmatrix} i_E \\ j_E \\ k_E \end{pmatrix}, \quad (1)$$

and transforming the gravity vector gk_E to body-axes, we have

$$\underline{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = g \begin{pmatrix} c\psi s\theta \\ -c\phi s\psi s\theta - s\phi c\theta \\ s\phi s\psi s\theta - c\phi c\theta \end{pmatrix}. \quad (2)$$

Note that unlike for conventional Euler angles sequence, equation (2) is a function of all Euler angles (Θ, Ψ, Φ).

Equations of Motion

Assuming xy and xz as planes of symmetry ($I_{xy} = I_{yz} = I_{xz} = 0$ and $I_{yy} = I_{zz}$), the 6-DOF rigid body equations of motion, referred to body axes can be written as

$$\begin{aligned} \sum X / m &= \dot{U} + QW - RV + g_x, \\ \sum Y / m &= \dot{V} + RU - PW + g_y, \\ \sum Z / m &= \dot{W} + PV - QU + g_z, \\ \sum L / I_{xx} &= \dot{P}, \\ \sum M / I_{zz} &= \dot{Q} - PR\eta, \\ \sum N / I_{zz} &= \dot{R} + PQ\eta, \end{aligned} \quad (3)$$

where $\sum X, \sum Y, \sum Z$ represent the sum of all forces, and $\sum L, \sum M, \sum N$ represent the sum of all moments along the body x, y and z axes, respectively. To equation (3) one must add the equations relating the Euler angle rates $\dot{\Theta}, \dot{\Psi}, \dot{\Phi}$, to the body axes angular rates P, Q, R :

$$\begin{aligned} \dot{\Theta} &= Q \frac{c\Phi}{c\Psi} - R \frac{s\Phi}{c\Psi}, \\ \dot{\Psi} &= Qs\Phi + Rc\Phi, \\ \dot{\Phi} &= P - Q \frac{c\Phi s\Psi}{c\Psi} + R \frac{s\Phi s\Psi}{c\Psi}. \end{aligned} \quad (4)$$

Note that the above transformation is singularity free for $\Theta = 90^\circ$, the singularity now appears for $\Psi = 90^\circ$. This singularity does not cause any problems for the vehicles of interest here.

For high-speed vehicles, the angular rate of the vertical relative to inertial space should also be taken into consideration.^{2,3} This effect may be incorporated by considering a pseudo-inertial frame x_E, y_E, z_E such that its vertical plane contains the equilibrium flight velocity and gravity vectors and the plane is coincident with the body pitch plane. The z_E axis keeps pointing along the vertical as the vehicle flies over a spherical earth. Then, the angular velocity of this pseudo-inertial frame relative to inertial space is

$$-\frac{\hat{U}}{\Re} j_E.$$

Using (1), expressing this angular velocity in the body-axes, we have

$$-\frac{\hat{U}}{\Re} \begin{pmatrix} s_\Psi \\ c_\Phi c_\Psi \\ -s_\Phi c_\Psi \end{pmatrix}, \quad (5)$$

where

$$\hat{U} = U c_\Psi c_\Theta + V(s_\Phi s_\Theta - c_\Phi s_\Psi c_\Theta) + W(c_\Phi s_\Theta + s_\Phi s_\Psi c_\Theta). \quad (6)$$

The three Euler angles relate the body axes to the pseudo-inertial frame x_E, y_E, z_E through (4). Knowing the angular velocity (5) of the pseudo-inertial frame with respect to inertial space, the total angular velocity of the body relative to inertial space may be written as,

$$\begin{aligned} P &= \dot{\Phi} + \dot{\chi} s_\Psi, \\ Q &= \dot{\Psi} s_\Phi + \dot{\chi} c_\Phi c_\Psi, \\ R &= \dot{\Psi} c_\Phi - \dot{\chi} s_\Phi c_\Psi. \end{aligned} \quad (7)$$

Linearized Equations of Motion

For linearization, we consider the total motion as consisting of the mean or equilibrium motion (denoted with the subscript 0) and perturbations about the mean (represented with small letters):

$$U = U_o + u, P = P_o + p, \Theta = \Theta_o + \theta, \text{ etc.}$$

The perturbations from the steady flight conditions are assumed to be so small that the products and squares of perturbations can be neglected and

$$c_\Theta \approx c_{\Theta_o} - \theta s_{\Theta_o}, s_\Theta \approx s_{\Theta_o} + \theta c_{\Theta_o}, \text{ etc.}$$

Assuming $P_o = R_o = V_o = 0$, we have the following first order perturbation equations

$$dX/m = \dot{u} + wQ_o + qW_o + \theta\Gamma_{X_\theta} - \psi\Gamma_{X_\Psi}, \quad (8)$$

$$dY/m = \dot{v} + rU_o - pW_o + \theta\Gamma_{Y_\theta} - \psi\Gamma_{Y_\Psi} - \phi\Gamma_{Y_\Phi},$$

$$dZ/m = \dot{w} - uQ_o - qU_o + \theta\Gamma_{Z_\theta} + \psi\Gamma_{Z_\Psi} + \phi\Gamma_{Z_\Phi},$$

$$\frac{dL}{I_{xx}} = \dot{p}, \quad (9)$$

$$\frac{dM}{I_{yy}} = \dot{q},$$

$$\frac{dN}{I_{zz}} = \dot{r} + pQ_o\eta.$$

where

$$\begin{aligned} \Gamma_{X_\theta} &= g(c_\Psi c_{\Theta_o}), \\ \Gamma_{X_\Psi} &= g(s_\Psi s_{\Theta_o}), \\ \Gamma_{Y_\Psi} &= g(c_\Phi c_\Psi s_{\Theta_o}), \\ \Gamma_{Y_\Phi} &= g(c_\Phi c_{\Theta_o} - s_\Phi s_\Psi s_{\Theta_o}), \\ \Gamma_{Y_\theta} &= g(s_\Phi s_{\Theta_o} - c_\Phi s_\Psi c_{\Theta_o}), \\ \Gamma_{Z_{\Phi, \Psi, \theta}} &= -\Gamma_{Y_{\Phi, \Psi, \theta}} (\Phi_o \rightarrow \Phi_o + \pi/2) \end{aligned}$$

Also, from (7)

$$\begin{aligned} p &= \left(\dot{\theta} - \frac{\hat{U}}{\Re} \right) s_\Psi c_\Phi + \dot{\chi} - \psi\Gamma_{p_\Psi}, \\ q &= \left(\dot{\theta} - \frac{\hat{U}}{\Re} \right) c_\Psi c_\Phi + \dot{\chi} s_\Phi + \psi\Gamma_{q_\Psi} - \phi\Gamma_{q_\Phi}, \\ r &= -\left(\dot{\theta} - \frac{\hat{U}}{\Re} \right) c_\Psi s_\Phi + \dot{\chi} c_\Phi + \psi\Gamma_{r_\Psi} - \phi\Gamma_{r_\Phi}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \Gamma_{p_\Psi} &= \dot{\chi}_o c_\Psi, \\ \Gamma_{q_\Psi} &= -\dot{\chi}_o c_\Phi s_\Psi, \\ \Gamma_{q_\Phi} &= \dot{\chi}_o s_\Phi c_\Psi - \dot{\Psi}_o c_\Phi, \\ \Gamma_{r_{\Psi, \Phi}} &= \Gamma_{q_{\Psi, \Phi}} (\Phi_o \rightarrow \Phi_o + \pi/2). \end{aligned}$$

and

$$\begin{aligned} \hat{U}_o &= U_o c_\Psi c_{\Theta_o} + W_o \Gamma_{\hat{U}_w}, \\ \hat{u} &= u c_\Psi c_{\Theta_o} + v \Gamma_{\hat{u}_v} + w \Gamma_{\hat{u}_w} + \theta \Gamma_{\hat{u}_\theta} + \psi \Gamma_{\hat{u}_\Psi} + \phi \Gamma_{\hat{u}_\Phi}, \end{aligned}$$

where

$$\begin{aligned} \Gamma_{\hat{u}_u} &= c_\Psi c_{\Theta_o}, \\ \Gamma_{\hat{u}_v} &= s_\Phi s_{\Theta_o} - c_\Phi s_\Psi c_{\Theta_o}, \\ \Gamma_{\hat{u}_w} &= \Gamma_{\hat{u}_v} (\Phi_o \rightarrow \Phi_o + \pi/2), \\ \Gamma_{\hat{u}_\Phi} &= -W_o \Gamma_{\hat{u}_v}, \\ \Gamma_{\hat{u}_\Psi} &= -U_o s_\Psi c_{\Theta_o} + W_o s_\Phi c_\Psi c_{\Theta_o}, \\ \Gamma_{\hat{u}_\theta} &= U_o \Gamma_{\hat{u}_u} (\Theta_o \rightarrow \Theta_o + \pi/2) \\ &\quad + W_o \Gamma_{\hat{u}_w} (\Theta_o \rightarrow \Theta_o + \pi/2). \end{aligned}$$

In the following, we develop a linearized model for reference flight conditions for which $\Theta_o = \Psi_o = 0$, and $\Psi_o = \Phi_o = 0$ so that the non-vanishing Γ 's are $\Gamma_{X_\theta} = \Gamma_{Y_\Phi} = g c_{\Theta_o}$, $\Gamma_{Y_\Psi} = \Gamma_{Z_\theta} = g s_{\Theta_o}$, $\Gamma_{p_\Psi} = \Gamma_{r_\Phi} = -\hat{U}_o / \Re$, $\Gamma_{\hat{u}_u} = c_{\Theta_o}$, $\Gamma_{\hat{u}_w} = s_{\Theta_o}$, $\Gamma_{\hat{u}_\theta} = -U_o s_{\Theta_o} + W_o c_{\Theta_o}$ and $\hat{U}_o = U_o c_{\Theta_o} + W_o s_{\Theta_o}$, $\hat{u} = u c_{\Theta_o} + w s_{\Theta_o} - \theta(U_o s_{\Theta_o} - W_o c_{\Theta_o})$. Thus, from (8) and (10):

$$\begin{aligned} dX/m &= \dot{u} + wQ_o + qW_o + \theta g c_{\Theta_o}, \\ dY/m &= \dot{v} + rU_o - pW_o - \psi g s_{\Theta_o} - \phi g c_{\Theta_o}, \\ dZ/m &= \dot{w} - uQ_o - qU_o + \theta g s_{\Theta_o}, \end{aligned} \quad (11)$$

$$\begin{aligned}
p &= \dot{\phi} - \psi \frac{\dot{U}_0}{\Re}, \\
q &= \dot{\theta} - \frac{\dot{u}}{\Re}, \\
r &= \dot{\psi} + \phi \frac{\dot{U}_0}{\Re}.
\end{aligned}$$

Note that if the rotation of the vertical is neglected, the relations between p, q, r and $\dot{\phi}, \dot{\theta}, \dot{\psi}$ are simplified to direct respective equalities. The rotation of the vertical involves some terms proportional to U_0 / \Re and W_0 / \Re , which are considered to be insignificant. Now applying all these assumptions, and simplifying the terms $W_0 q, Q_0 w, U_0 r, W_0 p, U_0 q, Q_0 u$, eqs. (9) and (11) reduce to

$$\begin{aligned}
dX / m &= \dot{u} + W_0 \dot{\theta} + C_{u\theta} \theta, \\
dY / m &= \dot{v} + U_0 \dot{\psi} - W_0 \dot{\phi} \\
&\quad + C_{v\phi} \phi + C_{v\psi} \psi, \\
dZ / m &= \dot{w} - U_0 \dot{\theta} - C_{w\theta} \theta, \\
dL / I_{xx} &= \ddot{\phi}, \\
dM / I_{yy} &= \ddot{\theta}, \\
dN / I_{zz} &= \ddot{\psi},
\end{aligned} \tag{12}$$

where,

$$\begin{aligned}
C_{u\theta} &= \frac{W_0 U_0}{\Re} S_{\theta_0} + g C_{\theta_0}, \\
C_{v\phi} &= \frac{U_0^2}{\Re} C_{\theta_0} + \frac{W_0 U_0}{\Re} S_{\theta_0} - g C_{\theta_0}, \\
C_{v\psi} &= \frac{W_0 U_0}{\Re} C_{\theta_0} - g S_{\theta_0}, \\
C_{w\theta} &= \frac{U_0^2}{\Re} S_{\theta_0} - \frac{W_0 U_0}{\Re} C_{\theta_0} - g S_{\theta_0}.
\end{aligned}$$

Now, for the perturbations due to aerodynamics forces and moments^{2,4,6,7}, we assume that the longitudinal aerodynamic forces (X, Z) and pitching-moment (M) are independent from lateral perturbed motions (v, p, r), we have $\{X, M, Z\}_{v, p, r} = 0$. Corollary of this assumption is that there can be no appreciable aerodynamic lateral force (Y), rolling-moment (P) and yawing moment (R) due to longitudinal perturbed motions (u, w, q). This implies that $\{L, Y, N\}_{u, w, q} = 0$. Thus, we have

$$\begin{aligned}
dX / m &= X_u(u - u_g) + X_w(w - w_g) + \Sigma X_{\delta} \delta, \\
dY / m &= Y_v(v - v_g) + Y_r(\dot{\psi} - r_g) + \Sigma Y_{\delta} \delta, \\
dZ / m &= Z_u(u - u_g) + Z_w(w - w_g) + \Sigma Z_{\delta} \delta, \\
dL / I_{xx} &= L_v(v - v_g) + L_p(\dot{\phi} - p_g) + \Sigma L_{\delta} \delta, \\
dM / I_{yy} &= M_u(u - u_g) + M_w(w - w_g) \\
&\quad + M_q(\dot{\theta} - q_g) + \Sigma M_{\delta} \delta, \\
dN / I_{zz} &= N_v(v - v_g) + N_r(\dot{\psi} - r_g) + \Sigma N_{\delta} \delta,
\end{aligned} \tag{13}$$

where δ represents a particular control surface deflection, $p_g = \partial w_g / \partial y$, $q_g = -\partial w_g / \partial x = -\dot{w}_g / U_0$ and $r_g = \dot{v}_g / U_0$. We have neglected all derivatives containing the rate of change of velocities except \dot{w} and \dot{v} .

Stability derivatives

Under the assumption taken in last section the only non-vanishing stability derivatives are: $\{X, M, Z\}_{u, w, q}$ and $\{L, Y, N\}_{v, p, r}$. These are derived as follows:

$$\begin{aligned}
\left(\frac{X_{u(w)}}{Z_{u(w)}} \right) &= \frac{1}{m} \frac{\partial}{\partial U \langle W \rangle} \hat{q} A \left(\frac{C_x}{C_z} \right) \Big|_0 \\
&= \frac{\rho A}{2m} \frac{\partial V_T^2}{\partial U \langle W \rangle} \left(\frac{C_x}{C_z} \right) \Big|_0 + \frac{\hat{q} A}{m} \frac{\partial}{\partial U \langle W \rangle} \left(\frac{C_x}{C_z} \right) \Big|_0.
\end{aligned} \tag{14}$$

Since, $C_{x,z} \equiv C_{x,z}(M, \alpha)$, therefore

$$\frac{\partial}{\partial U \langle W \rangle} \left(\frac{C_x}{C_z} \right) \Big|_0 = \frac{\partial M}{\partial U \langle W \rangle} \left(\frac{C_{x_M}}{C_{z_M}} \right) \Big|_0 + \frac{\partial \alpha}{\partial U \langle W \rangle} \left(\frac{C_{x_\alpha}}{C_{z_\alpha}} \right) \Big|_0.$$

Since for a symmetric flight ($V_0 = 0$),

$$V_T^2 = (U_0 + u)^2 + v^2 + (W_0 + w)^2,$$

and

$$\begin{aligned}
\frac{\partial V_T^2}{\partial U \langle W \rangle} &= 2U \langle W \rangle, \quad \frac{\partial V_T^2}{\partial v} = 2v, \\
\frac{\partial M}{\partial U \langle W \rangle} &= M \frac{U \langle W \rangle}{V_T^2}, \quad \frac{\partial M}{\partial v} = \frac{M}{V_T^2} v, \\
\frac{\partial \alpha}{\partial U \langle W \rangle} &= \frac{-W \langle +U \rangle}{U^2 + W^2}, \quad \frac{\partial \alpha}{\partial v} = 0, \\
\frac{\partial \beta}{\partial U \langle W \rangle} &= -v \frac{U \langle W \rangle}{V_T^2 \sqrt{U^2 + W^2}}, \\
\frac{\partial \beta}{\partial v} &= \frac{\sqrt{U^2 + W^2}}{V_T^2}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\left(\frac{X_{u(w)}}{Z_{u(w)}} \right) &= \xi U_0 \langle W_0 \rangle \left\{ \left(\frac{C_x}{C_z} \right) \Big|_0 + \frac{M_0}{2} \left(\frac{C_{x_M}}{C_{z_M}} \right) \right\}, \\
M_{u(w)} &= \zeta_{yy} L_{ref} U_0 \langle W_0 \rangle \left\{ C_m \Big|_0 + \frac{M_0}{2} C_{m_M} \right\} \\
&\quad - W_0 \langle +U_0 \rangle \frac{\zeta_{yy} L_{ref}}{2} C_{m_\alpha},
\end{aligned} \tag{15}$$

where,

$$\begin{aligned}
C_{x_M(z_M), m_M} &= \partial C_{x(z)} / \partial M \Big|_0, \\
C_{x_\alpha(z_\alpha), m_\alpha} &= \partial C_{x(z), m} / \partial \alpha \Big|_0, \\
C_{m_M} &= \partial C_m / \partial M \Big|_0, \\
C_{m_\alpha} &= \partial C_m / \partial \alpha \Big|_0.
\end{aligned}$$

Also,

$$\begin{aligned}
Y_v &= \frac{\xi}{2} V_{T_0} \frac{\partial C_y}{\partial \beta} \Big|_0, \\
\left(\frac{L_v}{N_v} \right) &= \frac{V_{T_0}}{2} \left(\frac{D_{ref} \zeta_{xx} \frac{\partial C_l}{\partial \beta}}{L_{ref} \zeta_{zz} \frac{\partial C_n}{\partial \beta}} \right) \Big|_0, \\
Y_{p(r)} &= \frac{\xi}{4} V_T L_{ref} \frac{\partial C_y}{\partial (p(r) L_{ref} / 2 V_T)} \Big|_0.
\end{aligned} \tag{16}$$

And

$$\begin{pmatrix} L_p \\ M_q \\ N_r \end{pmatrix} = \frac{V_{T_0}}{4} \begin{pmatrix} D_{ref}^2 \zeta_{xx} \frac{\partial C_l}{\partial (p D_{ref} / 2 V_T)} \\ L_{ref}^2 \zeta_{yy} \frac{\partial C_M}{\partial (q L_{ref} / 2 V_T)} \\ L_{ref}^2 \zeta_{zz} \frac{\partial C_n}{\partial (r L_{ref} / 2 V_T)} \end{pmatrix}_0, \quad (17)$$

$$\begin{pmatrix} L_r \\ N_p \end{pmatrix} = \frac{V_{T_0}}{4} \begin{pmatrix} D_{ref}^2 \zeta_{xx} \frac{\partial C_l}{\partial (r D_{ref} / 2 V_T)} \\ L_{ref}^2 \zeta_{zz} \frac{\partial C_n}{\partial (p L_{ref} / 2 V_T)} \end{pmatrix}_0.$$

Also, $V_{T_0}^2 = U_0^2 + W_0^2 \approx U_0^2$, for $W_0 \ll U_0$.

Simulation structure

State space model for decoupled longitudinal and lateral dynamics can be derived from equations (12) and (13). Longitudinal equations have u , w , θ and $\dot{\theta}$ as system states, control surface deflection δ is the input and system outputs are u , α and θ . Lateral dynamics are represented by states v , ψ , $\dot{\psi}$, ϕ and $\dot{\phi}$, control surface deflection δ is again the input and system outputs are taken as β , ψ and ϕ .

In this paper, simulation results are presented only for the lateral vehicle dynamics. Longitudinal vehicle dynamics can also be simulated in the same way.

The lateral dynamics of the vehicle are simulated in an LTV framework. In the LTV simulation, time varying vehicle properties are used. In order to make the simulation structure modular, the lateral dynamics are divided into three subsystems. These simulation blocks are shown as ϕ , β and ψ systems in Figure 1.

In the ϕ -system, input v is fed back from the β -system, δ is the control surface deflection, v_g is the wind gust disturbance, L_v , L_p and I_{xx} are time-dependent data taken from look-up tables. In this system $\dot{\phi}$ is computed according to the equation developed in the previous section and then $\dot{\phi}$ is integrated twice to get p and ϕ .

In the β -system, inputs ϕ and p are taken from ϕ -system, inputs ψ and r are fed back from ψ -system, δ is the control surface deflection, v_g is the wind gust disturbance, W_0 , $C_{v\phi}$, $C_{v\psi}$, Y_v , Y_r , m and U_0 are time continuous data.

In the ψ -system, input v is fed back from β -system, δ is the control surface deflection, v_g is the wind gust disturbance, N_v , N_r and I_{zz} are time continuous data taken from look up tables. In this system $\dot{\psi}$ is computed according to the equation developed in previous section and then it is

integrated twice to get r and ψ , which are also outputs of the system.

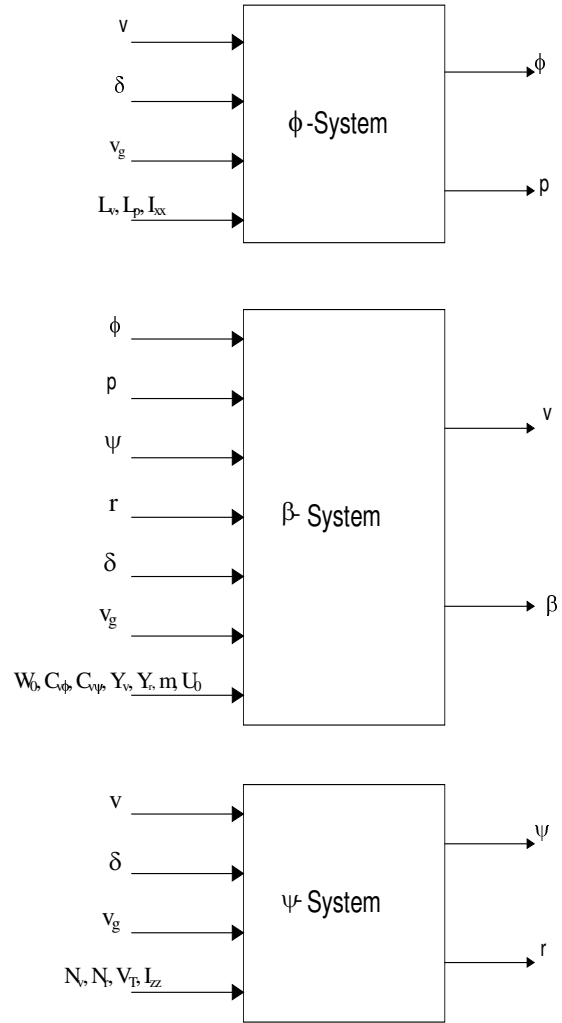


Figure 1: Simulation Blocks

Simulation results and discussion

Simulation results obtained from the LTV simulation are validated by comparison with the non-linear 6-DOF simulation. In figures 2 and 3, yaw and sideslip angles are compared for a given control surface deflection profile. In the first plot of figure 2, the given control surface deflection is shown. A 4° deflection is given in both directions. In the second plot, yaw angle ψ generated by the LTV simulation is compared with the yaw angle produced by 6-DOF simulation as a result of same control surface deflection profile. In the third plot of figure 2, the same comparison is done for sideslip angle β .

In figure 3, the same comparison is repeated for a different profile of control surface movement where a deflection of 4° in both directions is repeated twice.

In the first plot of figure 4, wind gust disturbance is used to compare the response of LTV simulation to that of the 6-DOF simulation. A wind gust of 20 m/sec is applied for 10 seconds. Yaw angle ψ and sideslip angle β are compared in second and third plots of figure 4.

In figure 5, the same comparison is repeated for a

different wind disturbance profile where a gust of 20 m/sec is applied in both directions. Comparison of Yaw angle ψ and sideslip angle β is again done in second and third plots of this figure.

From these results, it is observed that LTV simulation response is quite close to the response of the non-linear 6-DOF simulation for different control surface inputs and wind gust disturbances.

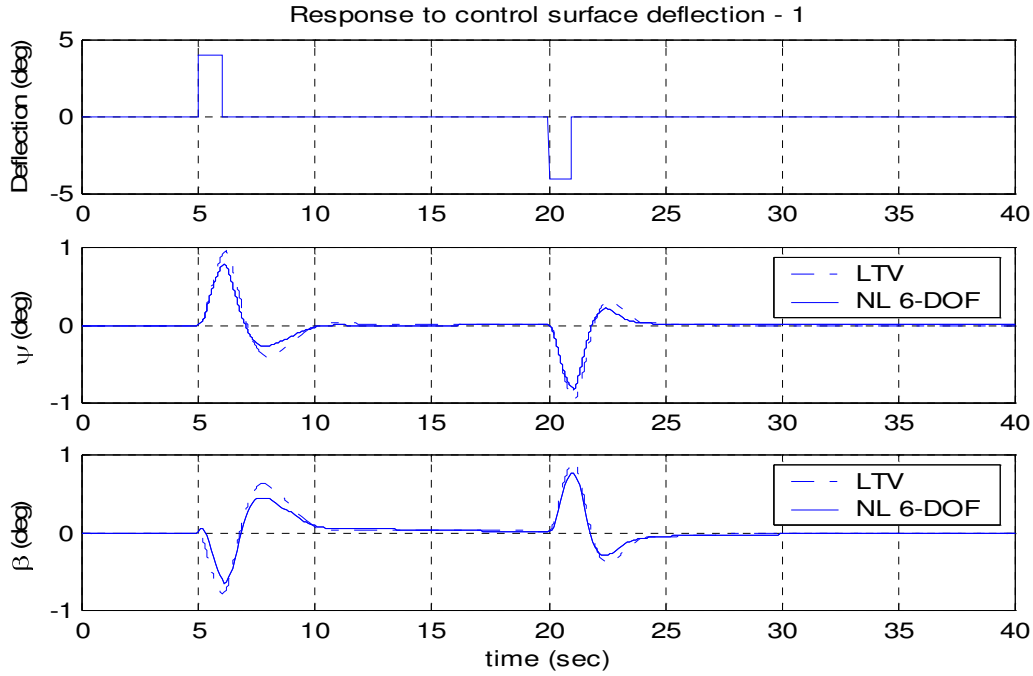


Figure 2: Comparison of ψ and β for control surface deflection – 1.

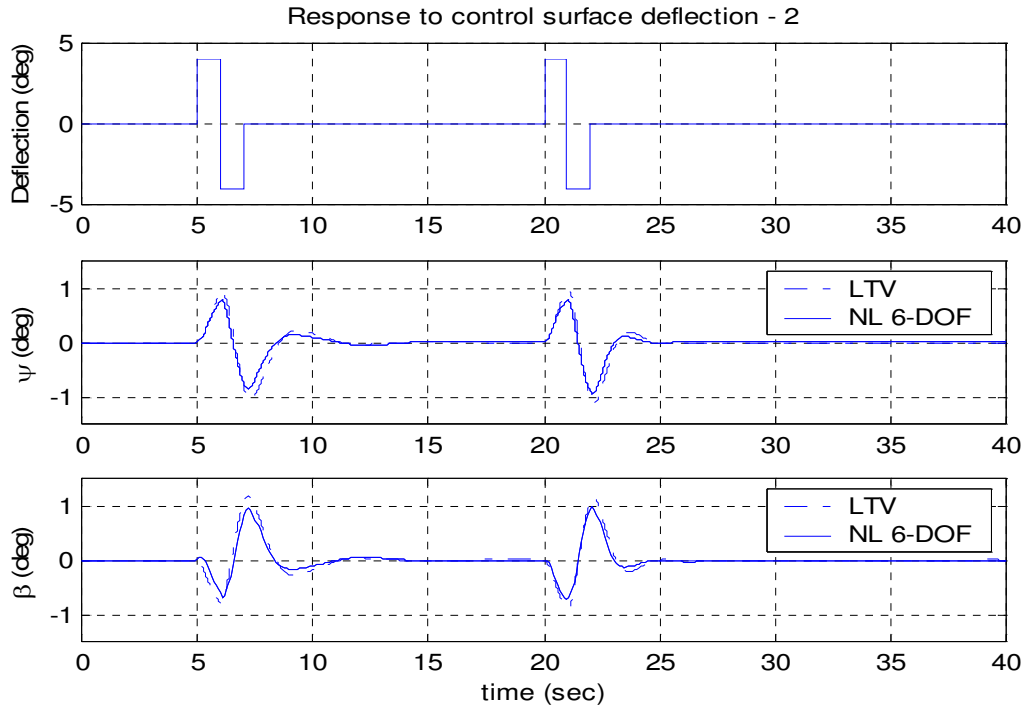


Figure 3: Comparison of ψ and β for control surface deflection – 2.

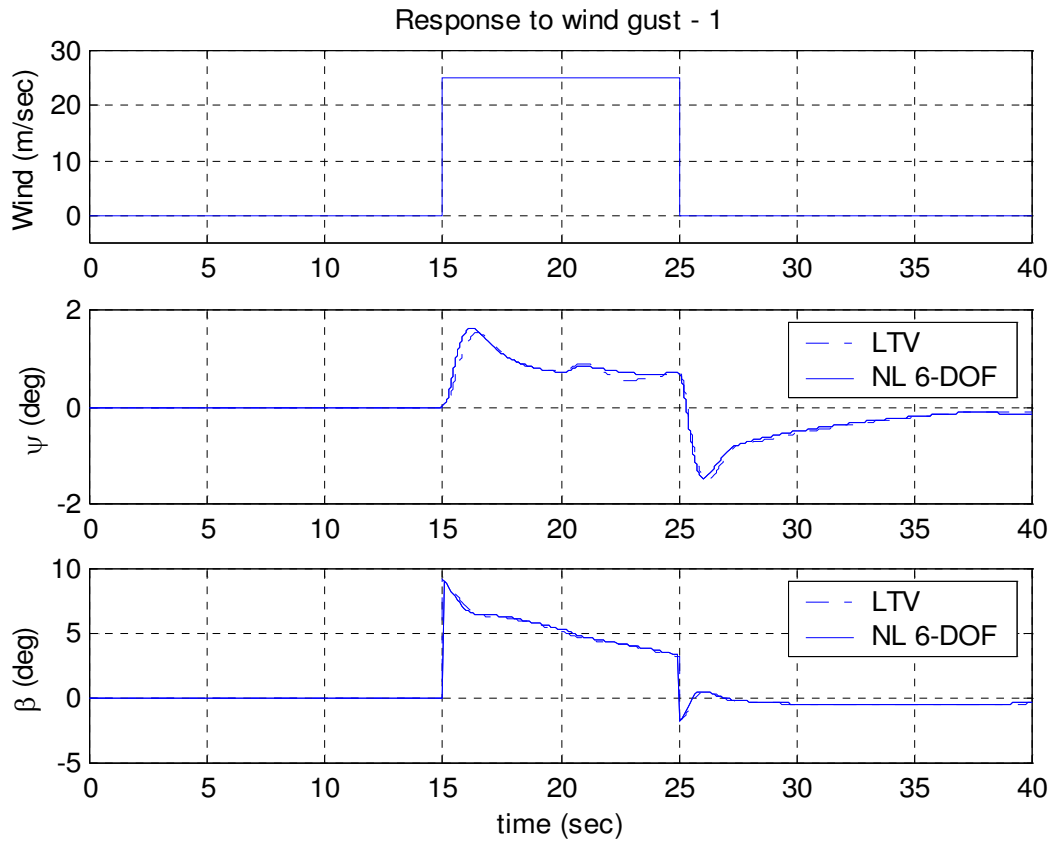


Figure 4: Comparison of ψ and β for wind gust disturbance - 1.

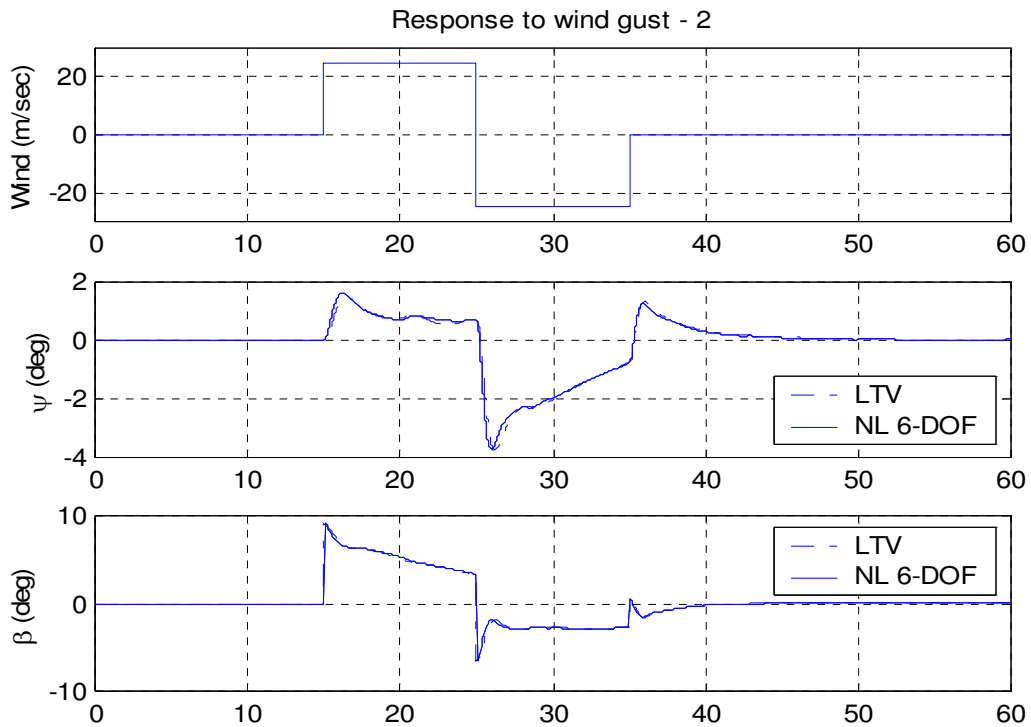


Figure 5: Comparison of ψ and β for wind gust disturbance - 2.

Conclusion

A linearized flight dynamics model is developed for a generic vertical lift-off high speed aerospace vehicle. Limitation of singularity in pitch is removed by changing the Euler angle sequence. The modified Euler angle sequence also simplifies the relationship between Euler angle rates and body-axes angular rates. Finally, a linear model is developed for a reference vehicle and it is shown through simulation that the dynamics of the Linear Time Varying system closely matches the dynamics of the corresponding nonlinear system.

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