Adaptive Sliding Mode Roll Control of a Canard-Controlled Missile

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Abstract: This work is meant to design a roll controller for a short range missile with a free-spinning tail using the sliding mode control technique. Four canards located near the nose of the missile provide aerodynamic control. Pitch/yaw stability is achieved through four rotating fins located at the tail of the missile. Range of the missile dictates the whole trajectory in the atmosphere where dynamic pressure varies between 0 and 7 bars. Also the missile can be fired at different launch angles, which implies that the profile of dynamic pressure is not fixed. Control strategy strongly depends on dynamic pressure in the case of canard-controlled missile. In the case of varying dynamic pressure, design of a robust roll controller that can keep roll angle near zero is a challenging task. In this paper, a roll controller is designed for a canard-controlled short range missile using sliding mode control technique. Adaption is done in some control parameters to cater the variation in dynamic pressure and different launch angle. The robustness of the roll controller is tested in the presence of aerodynamic and other disturbances. It is established that the proposed roll controller is capable of rejecting aerodynamic and other disturbances and keep the roll angle close to zero.
Key Words: Missile, sliding mode control, adaptive control, autopilot, aerodynamics, canards.

NOMENCLATURE

M	Mach number
α	Angle of attack [rad]
β	Side-slip angle [rad]
ϕ	Roll angle [rad]
θ	Pitch angle [rad]
ψ	Yaw angle [rad]
p	Rolling rate [rad/sec]
q	Pitching rate [rad/sec]
r	Yawing rate [rad/sec]
δa	Aileron deflection [rad]
Cl	Coefficient of rolling moment
I_X	Moment of inertia about body x-axis [kg-m^2]
I_Y	Moment of inertia about body y-axis [kg-m^2]
I_Z	Moment of inertia about body z-axis [kg-m^2]
ρ	Air density [kg/m^3]
q_∞	Dynamic pressure [Pascal]
S_{ref}	Reference area for aerodynamic coefficient
L_{ref}	Reference length for aerodynamic coefficient
SMC	Sliding mode control
Sign	Signum function
L	Rolling moment
INS	Inertial navigation system
Cl_P	$\frac{\partial Cl}{\partial p}$

1 INTRODUCTION

Typical autopilot design based on gain scheduling involves calculating control gains using linear techniques. This scheduling is done in different flight regimes via interpolation [1,2,4,10]. Such methods demands constant "tweaking" to be able to obtain best performance, specifically in the presence of nonlinearities. The performance of these methods greatly depends on the data quality of aerodynamic design inputs as well as nonlinearity in the system. Furthermore, performance degrades significantly in the presence of modelling error in conjunction with system nonlinearities [3].

In recent methods on-line estimation of plant parameters is involved to adjust the control law as parameters change and law is designed that is based on linear methods where the accuracy of parameter estimation cannot be guaranteed. Sliding Mode Control (SMC) also known as variable structure control is a robust control approach [1,3,4,5,8]. For the class of systems to which applies, sliding mode controller design provides a systematic approach to the problem that maintains and have consistent performance in the face of modeling imprecision. The controller so designed is unique since the performance of the controller depends on the design of sliding surface and not the states tracking directly [1]. Idea is to force the trajectory states towards the sliding surface and once achieved, the states are constrained to remain on the surface.

In order to guarantee the desired behavior of the closed loop system, the sliding mode controller requires infinitely fast switching mechanism. However, due to physical limitations in real world systems, directly applying the sliding mode control will always induce some oscillations in the vicinity of sliding surface so called chattering problem [1,4,5,8,9]. Chattering may be settled by smoothing the control input using boundary layer or bandwidth limited sliding mode

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control. Saturations and sigmoid functions are used, for example as “filters” for the output of a discontinuous signal in order to obtain a continuous one that is realizable by mechanical hardware. However, use of these functions carries with them an error penalty that must be acceptable to the designer [1,9].

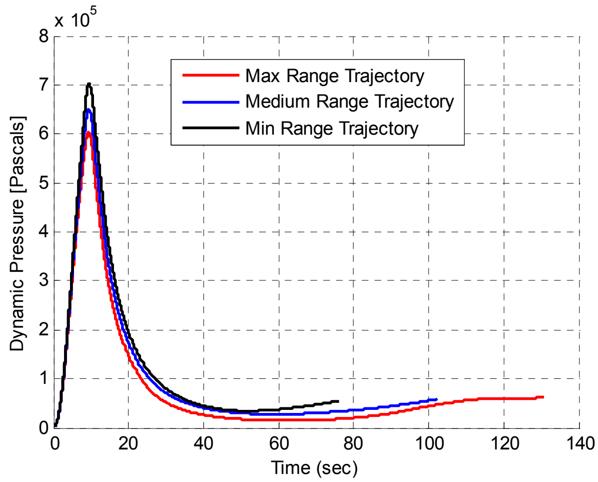

Figure 1: Dynamic Pressure for Different Trajectories

In figure-1, dynamic pressure variation is shown for different trajectories of a selected short range missile. Main challenge in the design of a roll control is the capability of controller to handle the variation of dynamic pressure during flight. In this paper, sliding mode control technique with adaption is used for roll control of a short range missile. Due to physical limitation of implementation, discontinuous sign function is replaced with an approximate continuous function. Adaption is done in some control parameters to handle the variation of dynamic pressure in a trajectory and also the change in profile of dynamic pressure in different trajectories. Robustness of the controller is tested for different trajectories with and without disturbances.

This paper has basically three parts. First part is problem formulation, which includes the basic configuration of selected missile, aerodynamic analysis and design specifications. Second part is control system design, which includes equations of motion, sliding surface design and design of final adaptive controller. Final part is simulation results and conclusion.

2 PROBLEM FORMULATION

In this section, a brief review of configuration, aerodynamic data and design specification is discussed.

2.1 Configuration

The missile understudy is a short range that is aerodynamically controlled with four fins (canards) near nose of the missile and four fins near tail of the missile provides the required pitch/yaw stability. The four fins near the nose of missile are connected with angular actuators and used for pitch, roll and yaw control. For roll control all fins are used since the missile assumes a cross-fin configuration. For pitch/yaw stability of missile, fins are necessary at the tail of missile [6,11]. Due to interaction of canard with rear-fins, rear fins are kept on a free rotating bearing. So rear fins can rotate freely along missile x-axis. As a result, free rotating rear fins provide almost the same pitch/yaw stability as fixed rear fins but there is no canard and rear fins interaction in case of free rotating rear fins [6].

The missile is powered by a rocket motor for early few seconds and the remaining flight is unpowered but aerodynamically controlled. Roll, pitch and yaw control is enable throughout the trajectory. Range of the missile dictates the whole trajectory is in the atmosphere and dynamic pressure varies between 0 to 7 bars.

2.2 Aerodynamic Analysis

The selected configuration is canard-controlled, fins at tail is necessary for pitch/yaw stability [6,7,12]. Before ruling out the possibility of fixed rear fins, some wind tunnel testing is performed with fixed rear fins that depict a phenomenon of aileron reversal as shown in figure-2. In figure-2, experimental rolling moment coefficient Cl is plotted versus aileron deflection δa for different mach numbers at α and β equal to zero. From figure-2; we can see that the slope $\frac{\partial Cl}{\partial \delta a}$ is positive for $\delta a = 0 \sim 5$ degrees and is negative for δa greater than 5 degrees. So the idea of fixed rear fins is not implementable and in our configuration, fins are fixed on a bearing that can rotate about body x-axis freely. In figure-3, experimental rolling moment coefficient Cl is plotted versus aileron deflection δa and we can see that there is no aileron reversal in the case of free rotating rear fins.

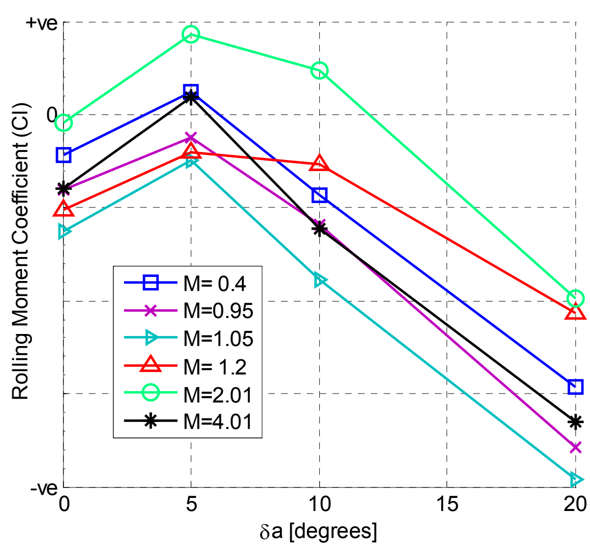

Figure 2: Cl vs δa for fixed-rear fins

From figure-3; we can also see that around transonic mach ~ 1, the aileron efficiency is higher as compare to subsonic and supersonic region. But overall we can see that Cl is almost linear vs δa, or in other words, the slope $\frac{\partial Cl}{\partial \delta a}$ is almost constant for all mach numbers.

The wind tunnel testing was done for only positive aileron deflections, so using symmetry we can compute Cl for negative aileron deflection also. Another main factor in the *Cl* is due to roll rate (damping). Also the available wind tunnel testing was only for positive aileron deflection, the rolling moment coefficient becomes

$$Cl = Cl(M, a, |\delta a|) * sign(\delta a) + Cl_P(M, \alpha) P \frac{L_{ref}}{V}$$

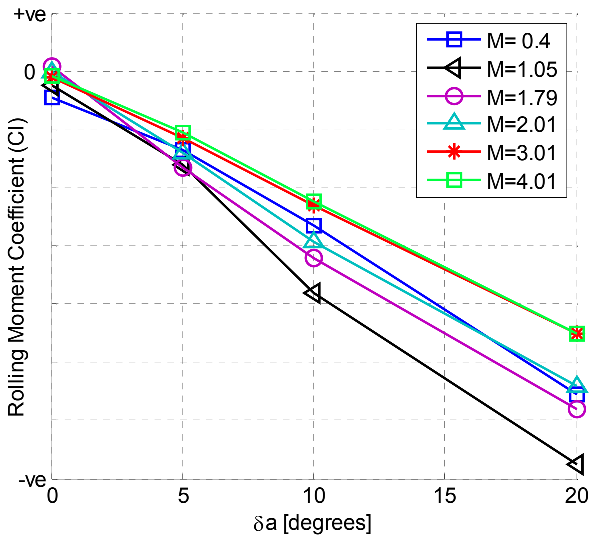

Figure 3: *Cl* vs δa for free rotating rear fins

In wind tunnel testing, it was also found that in the presence of α & β at different M produce a small rolling moment as compare to figure-3. So we add an extra term as a disturbance model in our aerodynamic roll model

$$Cl = Cl(M, a, |\delta a|) * sign(\delta a) + Cl_P(M, \alpha) P \frac{L_{ref}}{V} + Cl(\alpha, \beta, M) \qquad (1)$$

2.3 Design Specifications

In this paper, basic task is the design of a roll controller that can maintain roll angle near zero. For precise hitting of the missile, it is necessary to keep $|\phi| \leq 10°$ in the absence/presence of all disturbances. The installed sensor also has some limitations which imply that $|\dot{\phi}| \leq 50°/sec$. Different possible sources of disturbance are

- Thrust misalignment in body y-axis in the presence of thrust/CG offset in body z-axis or vice-versa.
- Aerodynamic asymmetries in rear fins or canards.
- Variation in moment of inertia.
- Roll moment due to angle of attack in the presence side-slip angle.
- Winds.

3 CONTROL DESIGN

In this section, detailed discussion on design of sliding surface, derivation of equivalent control and the adaption of different parameters is shown.

3.1 Equations of Motion

In this paper, we consider only the roll dynamics of the missile. Standard equations of motion for roll dynamics [2] are:

$$I_x \dot{P} + (I_z - I_y) qr = \sum L \qquad (2)$$
$$\dot{\phi} = P + \tan\theta \,(q \sin\phi + r \cos\phi)$$

Where $\sum L$ includes aerodynamics moments due to M, α, β, δa and P.

$$I_x \dot{P} + (I_z - I_y) qr = Cl\, q_\infty\, S_{ref}\, L_{ref} \qquad (3)$$
$$\dot{\phi} = P + \tan\theta \,(q \sin\phi + r \cos\phi) \qquad (4)$$

As we are assuming only roll dynamics here in this paper, so we assume that pitch and yaw rate is zero. Also our system is symmetric in pitch and yaw plane, so $I_z = I_y$. Equations (3-4) becomes

$$I_x \dot{P} = Cl\, q_\infty\, S_{ref}\, L_{ref} \qquad (5)$$
$$\dot{\phi} = P \qquad (6)$$

Substituting equation (6) in (5), we have

$$\ddot{\phi} = \frac{1}{I_x}\left(Cl\, q_\infty\, S_{ref}\, L_{ref}\right) \qquad (7)$$

Using equation (1) in equation (7), we have

$$\ddot{\phi} = \frac{q_\infty\, S_{ref}\, L_{ref}}{I_x}\Big(Cl(M, a, |\delta a|) * sign(\delta a) + Cl_P(M, \alpha)\dot{\phi}\frac{L_{ref}}{V} + Cl(\alpha, \beta, M)\Big) \qquad (8)$$

From figure-3, it is clear that *Cl* is almost linear versus δa, but its slope $\frac{\partial Cl}{\partial \delta a}$ depends on M and angle of attack. So equation (8) can be written as

$$\ddot{\phi} = \frac{q_\infty\, S_{ref}\, L_{ref}}{I_x}\Big(Cl_{\delta a}(M, a) * \delta a + Cl_P(M, \alpha)\dot{\phi}\frac{L_{ref}}{V} + Cl(\alpha, \beta, M)\Big) \qquad (9)$$

3.2 Sliding Surface

In sliding mode control design, first step is to design the sliding surface. The purpose of the switching control law is to drive the system state trajectory towards the user defined sliding surface also called switching surface. Ideally, once state trajectory intercepted the sliding surface, the switching control law maintains the system's state trajectory on the sliding surface for subsequent time.

In case of roll dynamics the state vector is $[p, \phi]^T$ and output of our interest is ϕ. So sliding surface become [1]

$$S = \left(\frac{d}{dt} + \lambda\right)\phi \qquad (10)$$

After simplification, our sliding surface becomes

$$S = \dot{\phi} + \lambda\phi \qquad (11)$$

3.3 Equivalent Control

Equivalent control is interpreted [1] as a continuous control law that would maintain $\dot{S} = 0$ if the dynamics were exactly known. In our case

$$\dot{S} = \ddot{\phi} + \lambda\dot{\phi}$$

Using equation (9) and $\dot{S} = 0$, we have equivalent control

$$\hat{\delta a} = \frac{1}{Cl_{\delta a}}\left(Cl_P(M,\alpha)\frac{L_{ref}}{V} + \frac{I_x\lambda}{q_\infty\, S_{ref}\, L_{ref}}\right)\dot{\phi} \qquad (12)$$

3.4 Final Adaptive SMC

The equivalent control can maintain the system at $\dot{S} = 0$ if the dynamics were exactly known but practically it is impossible. Therefore, some modifications in equivalent control must be made when the system to be controlled is uncertain,

$$\delta a = \hat{\delta a} - k sign(S)$$

After using equation (12), we have the total control law

$$\delta a = \frac{1}{Cl_{\delta a}}\left(Cl_P(M,\alpha)\frac{L_{ref}}{V} + \frac{I_x\lambda}{q_\infty\, S_{ref}\, L_{ref}}\right)\dot{\phi} - k\, sign(S) \qquad (13)$$

But this is a discontinuous function because of a *sign* function and cause the chattering problem. A reasonable approximation of *sign* function is $\frac{S}{|S|+\delta}$, where the δ term is chosen as a compromise between an ideal sliding motion and chattering. A small δ will give an ideal sliding motion with high chattering, on other hand, larger δ will give less chattering but a trajectory close to the sliding surface rather than remaining on it. So after using approximation of *sign* function, our total control law becomes

$$\delta a = \frac{1}{Cl_{\delta a}}\left(Cl_P(M,\alpha)\frac{L_{ref}}{V} + \frac{I_x\lambda}{q_\infty\, S_{ref}\, L_{ref}}\right)\dot{\phi} - k\,\frac{S}{|S|+\delta} \qquad (14)$$

In above equation, Cl_P is function of Mach and angle of attack. Due to weak dependence of Cl_P on Mach and absence of sensor for angle of attack measurement, Cl_P is kept zero in control design.

$$\delta a = \frac{1}{Cl_{\delta a}}\left(\frac{I_x\lambda}{q_\infty\, S_{ref}\, L_{ref}}\right)\dot{\phi} - k\,\frac{S}{|S|+\delta} \qquad (15)$$

But few points to be considered before fixing values of k, λ and δ are:

- As the missile can be fired for different range, which implies that initial launch angle and trajectory of the missile is not fixed.
- As the missile can be fired from different locations, which implies that for a same launch angle, the trajectory of the missile may be different.
- Dynamic pressure varies between 0 and 7 bars in one trajectory, which implies that canard will not generate the same rolling moment always.

From above point, it is very clear that k, λ and δ cannot be kept constant. So it is necessary that there must be some adaption in our control law as a function of dynamic pressure. For adaption, it is necessary to have dynamic pressure available in flight computer. As velocity and altitude of INS is available in flight computer, using velocity of INS and a reasonable model of density as a function of altitude we can easily compute the dynamic pressure.

Dynamic pressure varies too much during flight. During high dynamic pressure region, we are assuming larger disturbance so the value of δ is kept inversely proportional to the dynamic pressure. Similarly λ is directly proportional to the dynamic pressure, as disturbances in high dynamic pressure can generate big roll angle in very short time.

After adaption, the final controller takes the following form

$$\delta a = \frac{1}{Cl_{\delta a}}\left(\frac{I_x\lambda(q_\infty)}{q_\infty\, S_{ref}\, L_{ref}}\right)\dot{\phi} - k(q_\infty)\,\frac{S}{|S|+\delta(q_\infty)} \qquad (16)$$

4 SIMULATION RESULTS

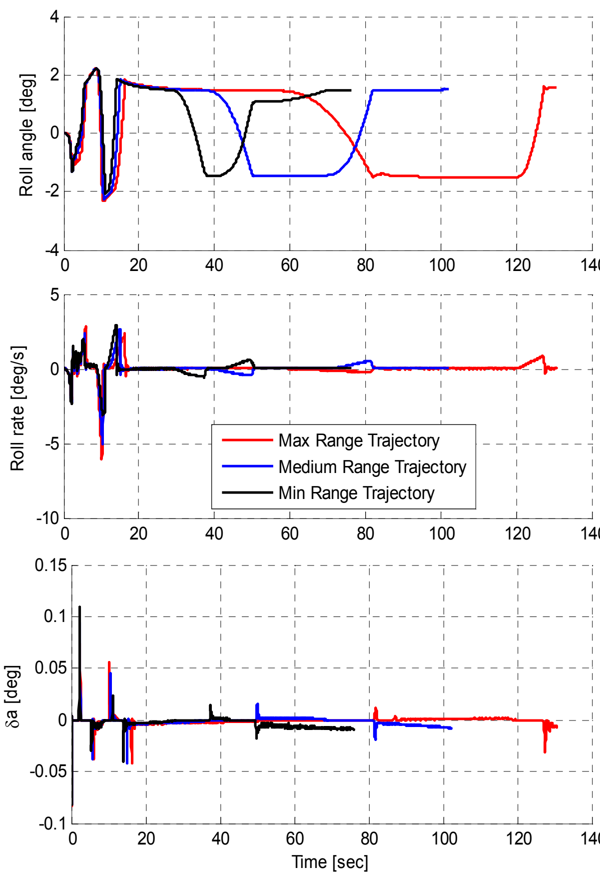

Figure 4: Simulation Results with No Disturbance

In figure-4, simulation results are shown for different trajectories in the absence of disturbances. In this figure, we can see the effect of variables that we neglect during control design phase. Overall contribution of neglected terms is very small and a maximum aileron deflection of ~0.1 degree is generated.

In figure-5, Simulation results are shown with aerodynamic disturbances. The effect of these disturbances reflects in the whole trajectory but dominant during maximum dynamic pressure region.

In figure-6, simulation results with all disturbances are shown. During powered phase, the maximum effect can be seen.

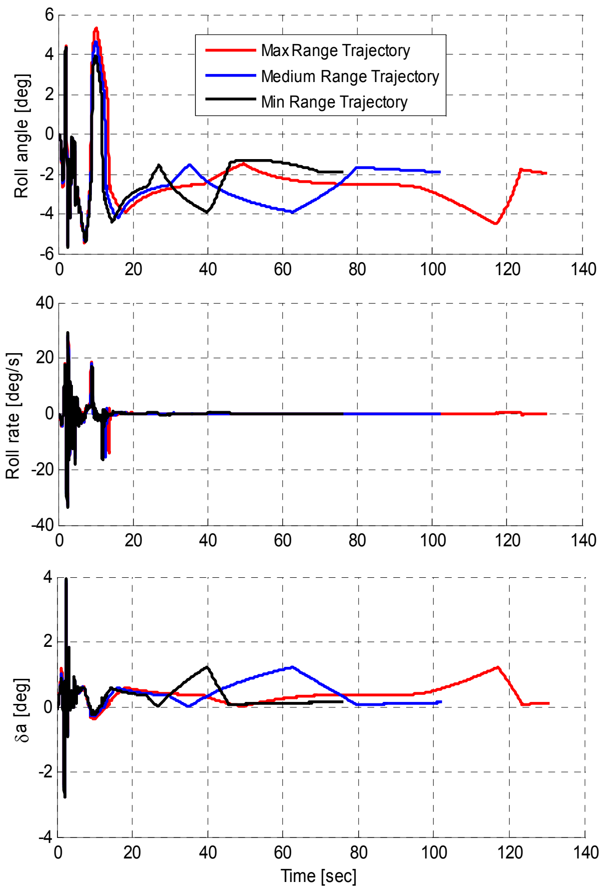

Figure 5: Simulation Results with Aerodynamic Disturbances

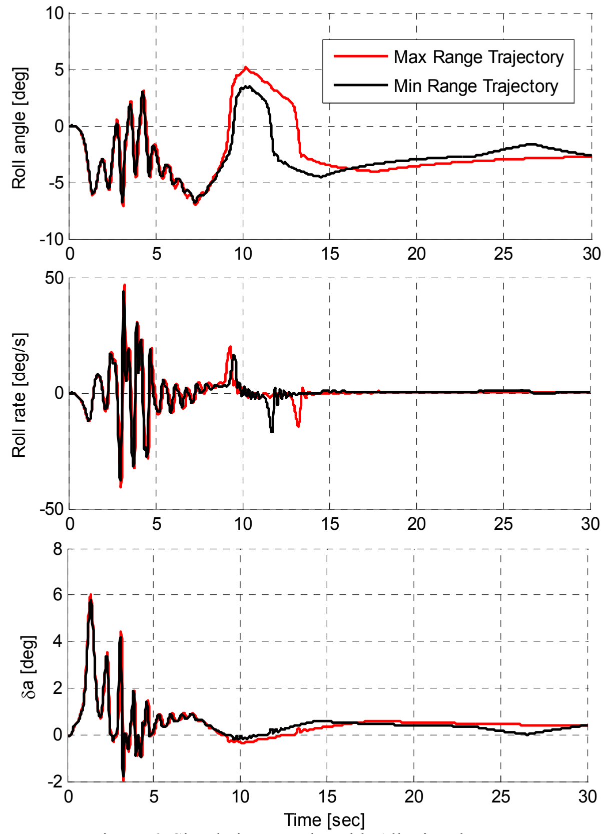

Figure 6: Simulation Results with All Disturbances

5 CONCLUSION

In figure-1 and 2, experimental rolling moment coefficient is plotted versus aileron deflection in case of fixed-tail and free-tail respectively. In the case of fixed-tail, canard interaction with rear fins badly affects the control authority. From these figures, it is clear that fixed-tail is not a viable solution in the case of canard controlled missile.

During control design, we neglected some terms and simplified some terms for simplicity of controller (e.g. Clp neglected, Cl vs δa assumed linear). The effect of these assumptions can be seen in figure-4. In case of simulation results with no disturbance, maximum roll angle is ~2 degrees, maximum roll rate is ~5 degrees/seconds and maximum aileron deflection is ~0.1 degrees. Very small control effort in figure-4, validate our assumptions during control design phase.

For robustness of the proposed controller, two cases with disturbance are shown in figure-5 & -6. As selected missile is an aerodynamically controlled missile therefore aerodynamic disturbance are also dominant disturbances and remains throughout the flight. In figure -5, simulation results are shown with aerodynamic disturbances only. In this case, maximum roll angle is ~6 degrees and maximum roll rate is ~ 30 degrees that meets our design specification criterion. Maximum control effort in this case is ~4 degrees that is well below our actuator limits.

In figure-6, simulations results are shown in the case of all possible disturbances. In this case, maximum roll angle is ~7 degrees, maximum roll rate is ~ 45 degrees/second and maximum control effort is ~6 degrees.

From figures-4, -5 & -6, it is clear that the proposed controller meets the required specification. Also we can see that a single proposed controller can work for all trajectories. Above mentioned results validate our claim of a robust controller that can keep roll angle near zero for all trajectories.

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